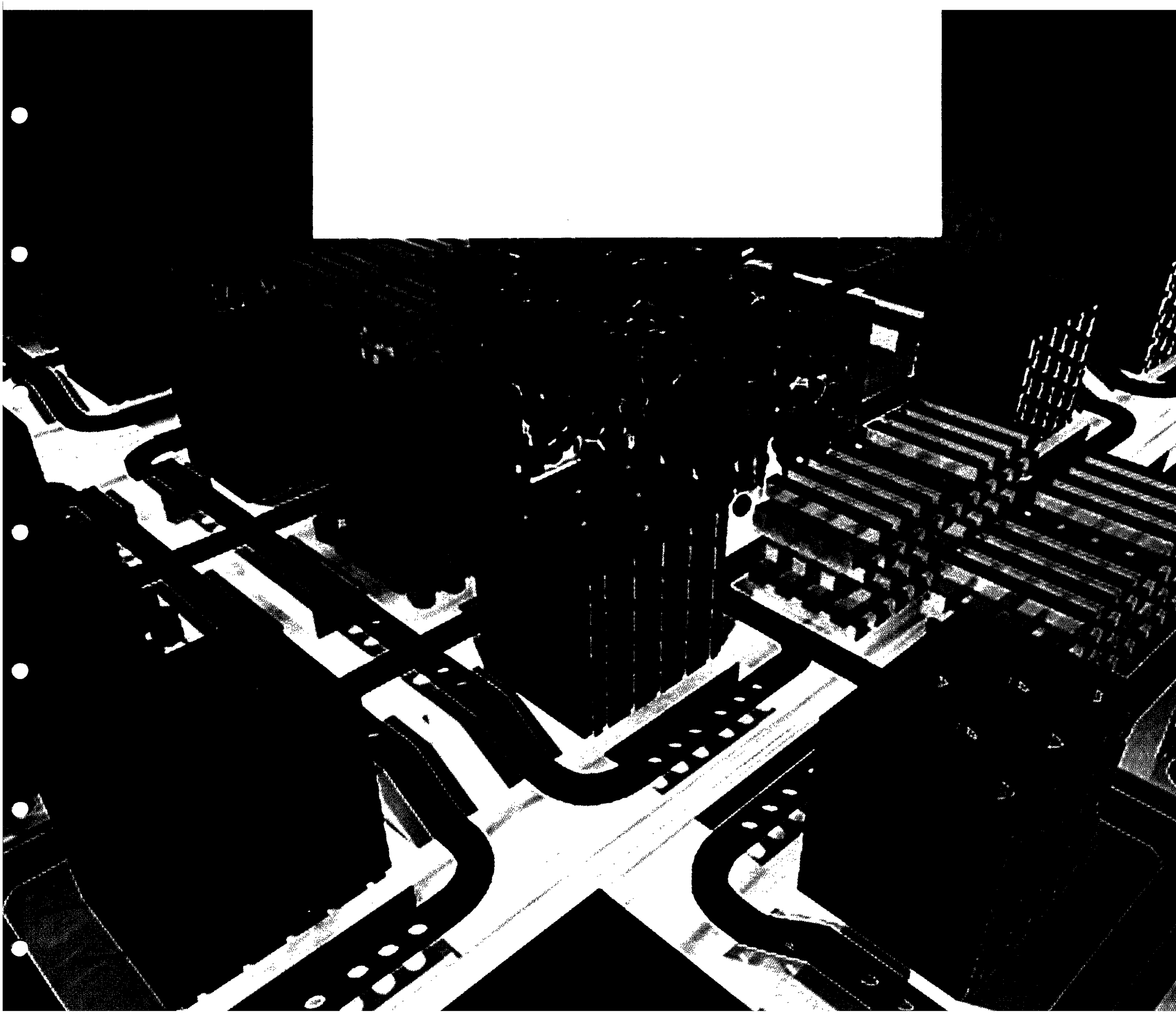
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

The RESEARCH LABORATORY *of* ELECTRONICS



Visual and Haptic Interactions in the Perception of Stiffness in Virtual Environments

By: Alberto J. Cividanes
and Mandayam A. Srinivasan

RLE Technical Report No. 640

June 2000

Visual and Haptic Interactions in the Perception of Stiffness in Virtual Environments

By: Alberto J. Cividanes
and Mandayam A. Srinivasan

RLE Technical Report No. 640

June 2000

Visual and Haptic Interactions in the Perception of Stiffness in Virtual Environments

by

Alberto J. Cividanes

Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degree of

Bachelor of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2000

Author ....
Department of Mechanical Engineering
May 5, 2000

Certified by ....
Mandayam A. Srinivasan
Principal Research Scientist
Thesis Supervisor

Accepted by ....
Ernest E. Cravalho
Professor of Mechanical Engineering
Chairman, Undergraduate Thesis Committee

Visual and Haptic Interactions in the Perception of Stiffness in Virtual Environments

by

Alberto J. Cividanes

Submitted to the Department of Mechanical Engineering on May 5, 2000, in partial fulfillment of the requirements for the degree of Bachelor of Science in Mechanical Engineering

Abstract

Psychophysical experiments were conducted in order to determine the impact of manipulated visual information and 3-D perspective on the haptic perception of stiffness in virtual environments. The Phantom force-reflecting haptic interface and a computer monitor were utilized to run experiments on the human discrimination of stiffness of two virtual springs. Ten subjects were asked to use the Phantom to compress two virtual springs sequentally and feel the corresponding displacement and forces through their hand, while they observed the visual deformation of the springs on a computer monitor. The subjects were asked to judge which spring was softer. Without the knowledge of the subjects, the visually presented deformation of each spring was manipulated systematically across experimental trials so that its relationship to the haptic stiffness of the spring was varied. Experiments were run on two different placement configurations of the springs. When the springs were placed side by side, only the effect of this manipulated visual information was studied. In a second configuration where the springs were placed one in front of the other, the effect of 3-D perspective was added as another factor. Results show that for both configurations, the percentage of correct responses decreased dramatically as the visual scaling parameter increased, demonstrating a visual-haptic illusion in stiffness perception. This suggests that visual information has a clear dominance over the kinesthetic hand position information in the discrimination of stiffness in this type of virtual environments. Thus, a proper combination of manipulated visual information and 3-D perspective can be used to enhance the range of haptic experience in multimodal virtual environments. Both the deliberately incorporated visual illusions and the ones created by 3-D perspective can be exploited in order to compensate for the limitations that current haptic interfaces have.

Thesis Supervisor: Mandayam A. Srinivasan
Title: Principal Research Scientist

Acknowledgements

One of the most difficult tasks of writing this thesis was to enumerate all the people that have helped with my research throughout the past year. First, I would like to thank my thesis and UROP advisor, Mandayam Srinivasan, who was given me his constant support and advice for the last year and a half. Besides introducing me to the world of haptics, he has been a great mentor who taught me the importance of conducting good scientific research.

I would also like to thank Wan-Chen Wu, a graduate student at the Touchlab. Without her help with the C++ program, this study could never have been done. Also, I want to thank her for her patience with me and for guiding me through all the minor details in this research experience. I also have to thank almost everyone at the Touchlab for making this such a nice experience. I cannot finish this section without mentioning those people who contributed with the data by participating as subjects: Maria Isabel Carnasciali, Hugo Delgado, Luis Miranda, Fernando Perez, Jeriel Rivera, David Antolin Laguna, Dolores Cruz, Suzie Mkandawire, Arin Basmajian, and Lizmarie Esparza. Finally, I wish to thank my family who supported me and provided me with the most basic tools to succeed in life. I hope you enjoy this thesis.

Table of Contents

1 Introduction 11
1.1 Purpose of the Study 12
1.2 Haptic Interfaces and Virtual Environments 12
2 Multisensory Research with Vision and Touch 15
3 Experiments 17
3.1 Apparatus 17
3.2 Procedure 18
3.2.1 Side - Side Configuration 20
3.2.2 Rear - Front Configuration 23
4 Results 25
5 Discussion 33
6 Conclusions 43
Appendix A Data Tables with Subject Responses 45
A.1 Subject Responses in Side - Side Configuration 45
A.2 Subject Responses in R - F Configuration: Front Spring Standard 49
A.3 Subject Responses in R - F Configuration: Rear Spring Standard 53
Appendix B Data Tables Apparent Stiffness of Springs 57
B.1 Apparent Stiffness in S-S Configuration 57
B.2 Apparent Stiffness in R-F Configuration (F Standard) 59
B.3 Apparent Stiffness in R-F Configuration (R Standard) 61
Bibliography 63

List of Figures

Figure 3.1: Phantom 1.5 17

Figure 3.2: Experimental setup used in both experiments 19

Figure 3.3: Side - Side configuration of virtual springs 20

Figure 3.4: Rear - Front configuration of virtual springs 23

Figure 4.1: Percent correct versus λ in S-S configuration 26

Figure 4.2: Percent correct versus λ in R-F configuration: Front spring standard 27

Figure 4.3: Percent correct versus λ in R-F configuration: Rear spring standard 28

Figure 4.4: Percent correct versus λ when $\Delta K/K_o$=25% 29

Figure 4.5: Percent correct versus λ when $\Delta K/K_o$=50% 30

Figure 4.6: Percent correct versus λ when $\Delta K/K_o$=75% 30

Figure 4.7: Percent correct versus λ when $\Delta K/K_o$=100% 31

Figure 5.1: Percent correct versus λ when $\Delta K/K_o$=25% 38

Figure 5.2: Apparent stiffness difference versus λ when $\Delta K/K_o$=25% 38

Figure 5.3: Percent of change in apparent stiffness difference when $\Delta K/K_o$=25% 38

Figure 5.4: Percent correct versus λ when $\Delta K/K_o$=50% 39

Figure 5.5: Apparent stiffness difference versus λ when $\Delta K/K_o$=50% 39

Figure 5.6: Percent of change in apparent stiffness difference when $\Delta K/K_o$=50% 39

Figure 5.7: Percent correct versus λ when $\Delta K/K_o$=75% 40

Figure 5.8: Apparent stiffness difference versus λ when $\Delta K/K_o$=75% 40

Figure 5.9: Percent of change in apparent stiffness difference when $\Delta K/K_o$=75% 40

Figure 5.10: Percent correct versus λ when $\Delta K/K_o$=100% 41

Figure 5.11: Percent correct versus λ when $\Delta K/K_o$=100% 41

Figure 5.12: Percent of change in apparent stiffness difference when $\Delta K / K_o$=100% .41

List of Tables

Table A.1: Percent correct versus λ when $\Delta K/K_o$=25% (S-S)..45

Table A.2: Percent correct versus λ when $\Delta K/K_o$=50% (S-S)..46

Table A.3: Percent correct versus λ when $\Delta K/K_o$=75% (S-S)..47

Table A.4: Percent correct versus λ when $\Delta K/K_o$=100% (S-S)......................................48

Table A.5: Percent correct versus λ when $\Delta K/K_o$=25% (F Standard).............................49

Table A.6: Percent correct versus λ when $\Delta K/K_o$=50% (F Standard).............................50

Table A.7: Percent correct versus λ when $\Delta K/K_o$=75% (F Standard).............................51

Table A.8: Percent correct versus λ when $\Delta K/K_o$=100% (F Standard)...........................52

Table A.9: Percent correct versus λ when $\Delta K/K_o$=25% (R Standard).............................53

Table A.10: Percent correct versus λ when $\Delta K/K_o$=50% (R Standard)...........................54

Table A.11: Percent correct versus λ when $\Delta K/K_o$=75% (R Standard)..........................55

Table A.12: Percent correct versus λ when $\Delta K/K_o$=100% (R Standard).........................56

Table B.1: Apparent stiffness of springs when $\Delta K/K_o$=25% in S-S................................57

Table B.2: Apparent stiffness of springs when $\Delta K/K_o$=50% in S-S................................57

Table B.3: Apparent stiffness of springs when $\Delta K/K_o$=75% in S-S................................58

Table B.4: Apparent stiffness of springs when $\Delta K/K_o$=100% in S-S..............................58

Table B.5: Apparent stiffness of springs when $\Delta K/K_o$=25% in R-F (F Standard)..........59

Table B.6: Apparent stiffness of springs when $\Delta K/K_o$=50% in R-F (F Standard)..........59

Table B.7: Apparent stiffness of springs when $\Delta K/K_o$=75% in R-F (F Standard)..........60

Table B.8: Apparent stiffness of springs when $\Delta K/K_o$=100% in R-F (F Standard)........60

Table B.9: Apparent stiffness of springs when $\Delta K/K_o$=25% in R-F (R Standard)......................61

Table B.10: AApparent stiffness of springs when $\Delta K/K_o$=50% in R-F (R Standard).................61

Table B.11: Apparent stiffness of springs when $\Delta K/K_o$=75% in R-F (R Standard)....................62

Table B.12: Apparent stiffness of springs when $\Delta K/K_o$=100% in R-F (R Standard)..................62

Chapter 1

Introduction

1.1 Purpose of the Study

The applications of haptic perception in virtual environments have increased dramatically during recent years. As virtual reality is increasingly used in the fields of entertainment, medicine, education and training, among others, a mere combination of visual and auditory feedback is not enough to provide a full virtual reality experience. In order to take full advantage of virtual reality, users must be able to touch, feel, and manipulate virtual objects in addition to hearing and seeing them. Thus, haptic displays with force and tactile feedback are necessary in order to improve the applications of virtual environments. It is imperative to understand that in order to be successful, a virtual environment does not need to perfectly replicate reality. Instead, it only needs to match the abilities and limitations of the human sensory, motor, and cognitive systems.

Intersensory research conducted throughout the years has revealed that visual information can alter the haptic perception of several spatial properties. As haptic interfaces are not as advanced as those interfaces that merely provide visual and auditory cues, a proper combination of visual and haptic cues can be used to enhance the impact of multimodal virtual environments on a user.

This thesis is an attempt to study the effects that deliberately incorporated visual illusions and 3-D perspective can have on the perception of stiffness in these environments. Psychophysical experiments have been conducted using the Phantom haptic interface, in which subjects have been asked to discriminate between the stiffness of two virtual springs. It is believed that a proper combination of manipulated visual information and 3-

D perspective can be exploited in order to compensate for the limitations that current haptic interfaces have today in terms of bandwidth, resolution, and workspace.

1.2 Haptic Interfaces and Virtual Environments

At the pace at which computer technology is currently growing, visual and auditory feedback alone cannot provide a person with a full experience in a virtual environment. Haptic interactions with the virtual objects are essential in this type of environments, as being able to touch, feel, and manipulate objects in virtual environments provides a sense of immersion in the environment that is not possible with only visual and auditory cues. A better immersion in a virtual environment can be achieved by the synchronous operation of a visual and auditory display with a haptic interface.

Haptic interfaces enable manual interaction with virtual environments or teleoperated remote systems. These devices are employed for tasks that are typically performed by using the hands, primarily manual exploration and manipulation of objects [11]. These manual interactions may also be accompanied by the stimulation of additional sensory modalities such as vision and audition. As human users perform tasks with a haptic interface, desired motor actions are conveyed by physically manipulating the interface, which displays tactual sensory information to the user by appropriately stimulating his or her tactile and kinesthetic sensory subsystems. Typically, force-reflecting haptic interfaces perform two basic functions: (1) To measure the positions, as well as their time derivatives, of the users' hands and (2) to display contact forces to the user.

The haptic interface utilized in this experiment was the Phantom version 1.5. The use of this device, developed by Sensable Technologies, has started a new field analogous to computer graphics called computer haptics. This new field is primarily related to the techniques and processes associated with generating and displaying synthesized haptic stimuli to the human user [12].

The Phantom interface allows motion along six degrees of freedom and measures this motion. In addition, it can exert controllable forces on the user along three of those degrees of freedom. To evoke the sensation of touching objects, geometric, material, kinematic, and dynamic properties need to be modeled of the world that is desired to be represented. Rendering computer programs for the Phantom typically run at 500 to 2,000 Hz, and it is important that these rendering algorithms are efficient.

Chapter 2

Multisensory Research with Vision and Touch

Previous psychophysical research has been conducted studying the intersensory relationship between vision and touch. Evidence based on previous experiments shows that visual information can alter the haptic perception of several spatial properties. Welch and Warren have written an extensive review of experiments done in the past concerning intersensory interactions, particularly visual and haptic interactions [16].

There are a few studies that are directly related to the one presented in this thesis. The first one is a preliminary study conducted by Srinivasan, Beauregard, and Brock in 1996 that studied the impact of visual information on the haptic perception of stiffness in two dimensional virtual environments [13]. By using a three degrees of freedom, force-reflecting haptic interface called the Planar Grasper, three subjects were asked to discriminate between the stiffness of two virtual springs. The results were very consistent, and they concluded that subjects tended to rely more on visual rather than haptic cues for judging displacements. This behavior occurred even when the visual deformation of the springs did not match their actual haptic deformation. Thus, they suggested the possibility that manipulated visual information can be used to improve haptic perception experiences in virtual environments.

In 1997, Durfee et al. conducted a similar experiment using a rotary electric motor as the haptic interface [2]. Seven subjects were required to discriminate between the stiffness of two virtual springs that had a mismatch between their visual and haptic stiffness. The results show that when subjects made errors in haptic estimation, the errors tended to follow the visual cues. This happened particularly for a large mismatch of visual and haptic stiffness.

Another important study is closely related to the one presented in this thesis. An experiment conducted by Srinivasan, Wu, and Basdogan in 1999 studied the effect of 3-D perspective vision on the perception of size and object stiffness in virtual environments [18]. In this case, there was no deliberately introduced mismatch between visual and haptic stiffness of the springs. This study is very important to the one on this thesis, as the same virtual environment and experimental setup was used in both subjects, with the exception that they did not manipulate the mismatch between visual and haptic stiffness. They concluded that the 3-D perspective visual displays generated visual and haptic illusions, such that farther objects are perceived to be shorter in length when there are only visual cues present, and softer when there are only haptic cues presence. However, when both sensory cues were provided, sensory data was fused such that vision and touch compensated for the bias due to each other.

Chapter 3

Experiments

3.1 Apparatus

A haptic device called the Phantom (Personal Haptic Interface Mechanism) version 1.5 was used in these experiments in order to render the haptic environment. The Phantom is a three axis force reflecting device that provides six degrees of freedom; three active and three passive (Figure 3.1). A stylus, which is a pen-like end effector, is held by the user and manipulated to explore a 19cm x 27cm x 38cm haptic workspace. The maximum exertable force with the Phantom is 8.5N. It has a maximum closed stiffness of 3.5 N/mm, while the backdrive friction is 0.04N. The nominal position resolution for this haptic device is about 0.03mm, and the inertia, or the apparent mass at the tip, is always less than 75g for any given configuration.

Figure 3.1: Phantom 1.5.

The graphic virtual environment was created using the OpenInventor computer graphics software. An SGI workstation (Indigo 2, 64 bit system and ISA interface card) was connected to the haptic device through a 12-bit D/A and a 19in monitor was used to show the visual environment. The monitor had a high resolution of 1280 x 1024. Both haptic and graphic environments were synchronized and controlled by means of C programming. The position and orientation of the tip of the stylus were sensed during the exploration process, and a corresponding force signal was sent back to the device to construct the virtual objects specified by the computer program. The graphic display had an updating rate of approximately 30 frames/s, while the haptic display had a rate of about 1 kHz. Both of these rates exceeded the perceptual fusion frequencies for the human eye and hand, respectively.

3.2 Procedure

Ten subjects between the ages of 20-22 years old participated in the experiments. They were all undergraduate students at the Massachusetts Institute of Technology and were paid on an hourly basis. Exactly five males and five females participated in the study, all of which were right handed with no known hand disorders. All subjects used their right hand for the experiments.

Two experiments were designed in order to investigate the effects of manipulated visual information and 3-D perspective on the haptic perception of stiffness in virtual environments. In both cases, the subjects were asked to sit comfortably on a chair located at a fixed distance from the computer and the haptic device (Figure 3.2). Two rectangular blocks with slots on top to prevent the tip of the Phantom from slipping, were displayed to the user and represented two virtual springs. By holding the stylus of the Phantom haptic device, subjects were asked to press on the virtual blocks, as they saw on the monitor what they were perceiving haptically with the Phantom. They were asked to discriminate between the stiffness of the two virtual "springs" and answer to the question: "Which button is softer?". In the first experiment, the virtual blocks were placed side by side (S-S configuration), and only the effect of manipulated visual information on the perception of stiffness was studied. In the second experiment, the blocks were arranged in a Rear - Front configuration (R-F configuration), where one was placed right in front of the other. In this case, both the effects of visual information and 3-D perspective were studied. In both experiments, a small black curtain was placed between the Phantom haptic interface and the user so that the subjects could not readily observe the position of their hands.

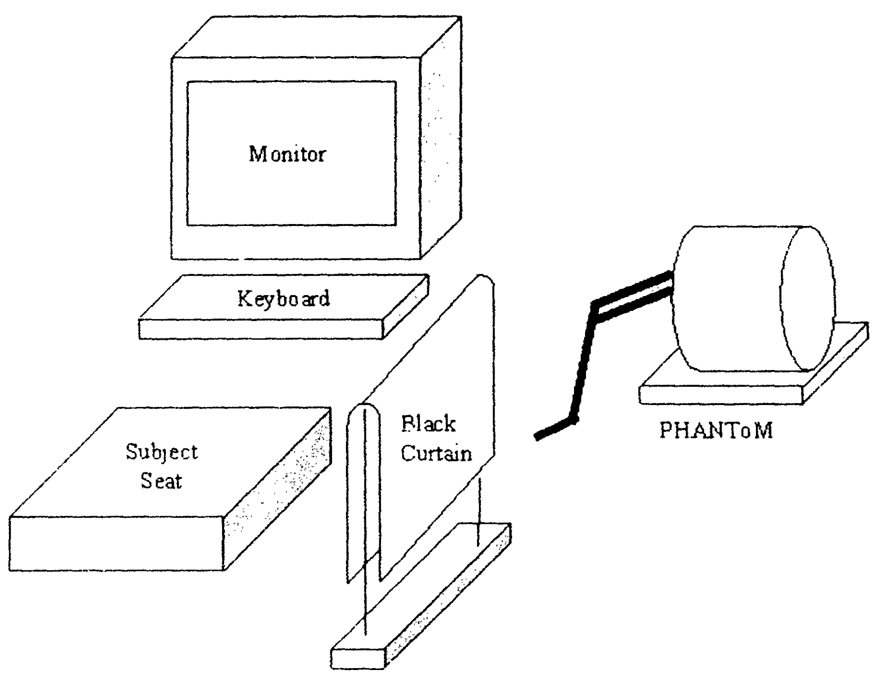


Figure 3.2: Experimental setup used in both experiments.

3.2.1 Side - Side Configuration

In the S-S configuration, the two virtual springs shown as blocks were placed side by side along the horizontal axis on the monitor screen and displayed to the subject. Figure 3.3 shows the actual screen that the subjects saw when the experiment was performed. The subjects held the stylus of the Phantom and pressed the blocks within the slots at the top while seeing what they touched on the monitor screen. A visual cursor showing the position of the stylus tip was displayed visually on the screen in order to help the subject navigate through the 3-D virtual environment.

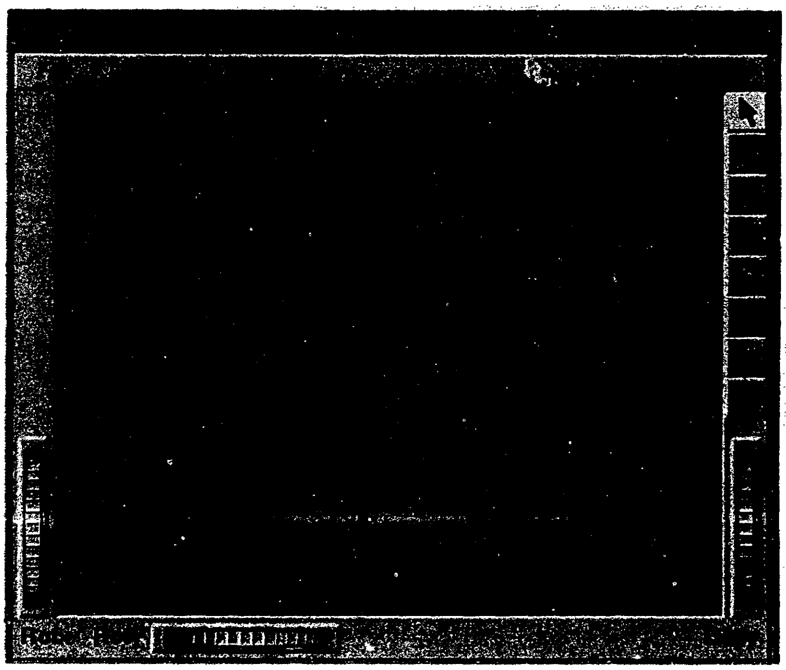


Figure 3.3: Side - Side configuration of the virtual "springs".

As the subjects explored the virtual springs and felt the stiffness of each spring, they were asked to observe the image on the screen that showed the corresponding spring deformations. They were allowed to compress the springs as many times as they wanted, and when they were finished they were asked to select which one of two springs felt softer by answering to the question: “Which button is softer?”. Subjects responded by typing “1” for the left spring and “2” for the right spring. It should be noted that the subjects received no performance feedback as they performed the experiment.

In each trial, the stiffness of one of the two springs was always equal to a reference stiffness, $K_o = 0.25$N/mm. The stiffness of the other spring was always higher, the reference stiffness plus an increment $(K_o + \Delta K)$. This ΔK increment was either 25%, 50%, 75%, or 100% of the reference stiffness. Trials were randomized so that both springs had an equal probability of having the reference stiffness, and the subject could not have prior knowledge of which spring was softer.

During the experiment, visual information was manipulated so that the visual deformation of the springs that the subject saw on the screen did not always correspond to the actual or haptic deformation of the springs that they were sensing with their hand with the aid of the Phantom. The discrepancy between the haptic deformation of the spring and the visually displayed spring deformation was varied systematically, but was randomized across the trials. It ranged from zero discrepancy, that is, that the visual deformation of each spring corresponded to its haptic deformation, to a case of a completely interchanged discrepancy, where the visual deformation of the softer spring was equal to the visual deformation of the harder spring for that same force and vice versa. The relationship between the haptic and visual deformation is given by the following set of equations:

$$x_{h,\,ref} = \frac{F}{K_o} \tag{3.1}$$

$$x_{v,\,ref} = \frac{F}{(1-\lambda)K_o + \lambda(K_o + \Delta K)} \tag{3.2}$$

$$x_{h,\,comp} = \frac{F}{(K_o + \Delta K)} \tag{3.3}$$

$$x_{v,\,comp} = \frac{F}{(1-\lambda)(K_o + \Delta K) + \lambda K_o} \tag{3.4}$$

where $x_{h,ref}$ and $x_{v,ref}$ are the haptic and visual displacements, respectively, for the spring with the standard reference stiffness. The relations for $x_{h,comp}$ and $x_{v,comp}$ give the haptic and visual displacements, respectively, for the comparison or stiffer spring (that is, the spring with the reference stiffness plus the increment). The visual scaling parameter is given by λ.

It can be observed from equations 3.1 and 3.3 that actual spring deformations for a given force are simply equal to the force divided by the stiffness of the spring. On the other hand, equations 3.2 and 3.4 show that the spring displacements that are visually displayed on the monitor screen are equal to the applied force divided by a weighted average of both spring constants. The influence of each spring stiffness constant depends of the scaling factor, λ. In the experiment, λ was varied from 0 to 1 at intervals of 0.25. When λ equals 0, the actual and visual deformations for each spring are the same. Increasing values of λ skews visual information so that the softer spring is visually compressed less than it should. For the case when λ equals 0.5, both springs will thus have an identical visual deformation, even though their stiffness are not equal. However, when λ equals 1 the visual displacement of each spring is determined only by the spring constant of the other spring.

For this experiment, there were a total of 20 possible combinations of the four values of increments in stiffness and the five values of λ. Each experimental session consisted of 40 trials, that is, two trials for each possible combination. For each of the two trials per case in every experimental session, the reference stiffness was contained by a different spring. All the trials within a session were randomized, so no two experimental sessions had the trials in the same particular order. For the S-S configuration, each subject was tested on ten different experimental sessions, giving a total of 400 trials per subject (20 trials for each ΔK and λ value pair).

3.2.2 Rear - Front Configuration

The experiment for the S-S configuration was repeated with a different configuration of the virtual springs. In this case, the springs were positioned one in front of the other on a rectangular block along the z-axis, so the spring in the front looked significantly larger than the one in the back. Figure 3.4 shows the actual screen that the subjects saw when the experiment was performed.

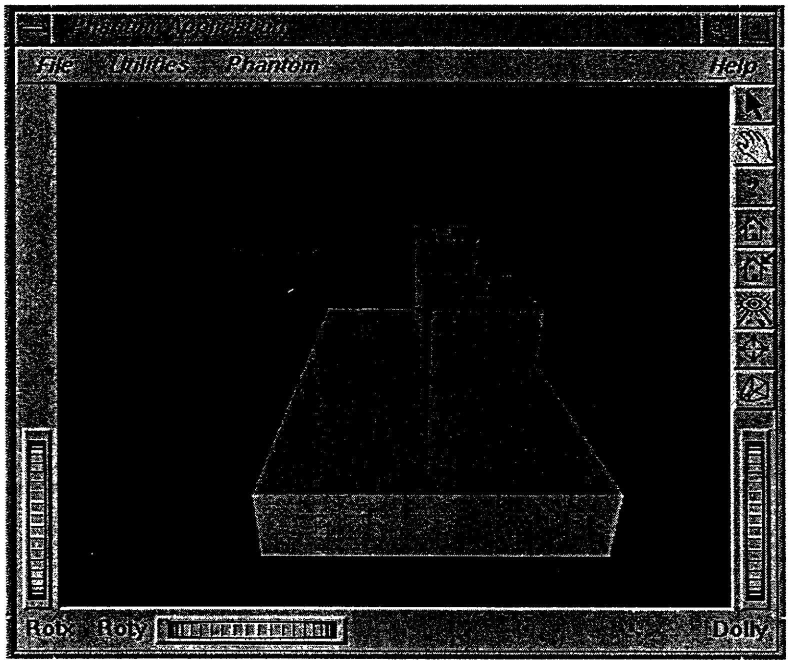


Figure 3.4: Rear - Front configuration of the virtual "springs".

In this case, the additional parameter of 3-D perspective was added to the experiment. Just as in the experiment with the S-S configuration, the manipulated visual deformations of the springs are still governed by equations 3.1 to 3.4.

For the R-F configuration, trials were arranged in a similar manner as in the S-S case. However, since 3-D perspective is a new factor that needed to be taken into account, twice the amount of data sets were displayed to each subject. Thus, each subject had 400 trials with the rear spring as the standard, and another 400 trials with the front spring as standard. The total of 800 trials per subject were arranged in 20 experimental sessions of 40 trials each. The S-S and R-F configuration experiments were alternated for every subject. For every trial, the subject's response was recorded in order to compare it with the correct answer to the question posed.

Chapter 4

Results

The results from the experiments show that manipulated visual information has a great impact on the perception of stiffness. Figures 4.1 to 4.3 show the results of both experiments graphically as plots of percent correct responses versus the visual scaling parameter λ. The results plotted are the averages across the ten subjects for the S-S configuration, and for the R-F configuration with either the front or the rear spring as standard. The results have also been tabulated numerically in Appendix A.

Figure 4.1 shows the results for the S-S configuration as percent of correct responses as a function of λ for all the values of ΔK. For the different cases of ΔK, the results are very consistent when $\Delta K/K_o$ is 50%, 75%, and 100%. However, for the case in which $\Delta K/K_o$=25%, a more linear behavior can be observed.

The graph presented in figure 4.1 shows that for the S-S configuration, when λ is zero and the discrepancy between visual and haptic spatial information was nonexistent, subjects responded extremely well, getting between 90% to 98% of their responses correct. These values approximate very closely the expected results. As λ was increased, however, the performance decreased for all subjects almost linearly as expected. At λ=0.5, the percentage of correct responses ranged between 57% to 65%. This was the case when both springs had exactly the same visual deformation for a given force. In the other extreme case, when λ was equal to 1, the subjects responded correctly only in 12% to 20% of the trials.

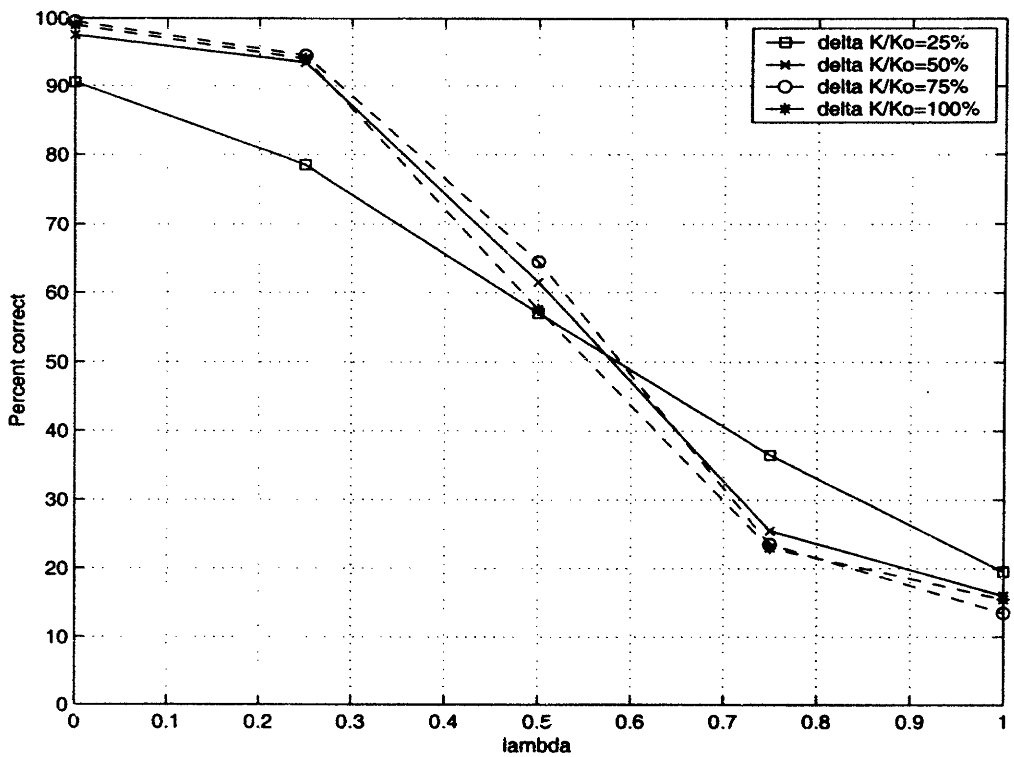


Figure 4.1: Percent correct versus λ in S-S configuration

Similar results have been plotted for the experiments in the R-F configuration. Figures 4.2 and 4.3 show graphs in the form of percentage of correct responses versus the visual scaling parameter λ for the different cases of ΔK. Figure 4.2 shows the results for the case when the front spring was the standard; that is, it contained the reference stiffness K_o. Figure 4.3 shows a similar graph for the R-F configuration, when the rear spring was the standard.

It can be observed from the graph presented in figure 4.2 that when the front spring was the standard in the R-F configuration, the results show a similar behavior that in the S-S configuration. When λ=0, subjects responded correctly between 93% to 98% of the cases. As λ was increased, however, the performance decreased for all subjects as expected. At λ=0.5, the percentage of correct responses ranged between 63.5% when

$\Delta K/K_o$=25% to 71% when $\Delta K/K_o$=100%. At this point, both springs had exactly the same visual deformation for a given force, and the difference in the visual perception was due to the 3-D perspective. When λ was equal to 1, the subjects responded correctly from 12% of the trials when $\Delta K/K_o$=100% to 34.5% of the cases when $\Delta K/K_o$=25%. Again, as in the S-S configuration, the results were very similar for all different values of ΔK, except when $\Delta K/K_o$=25%.

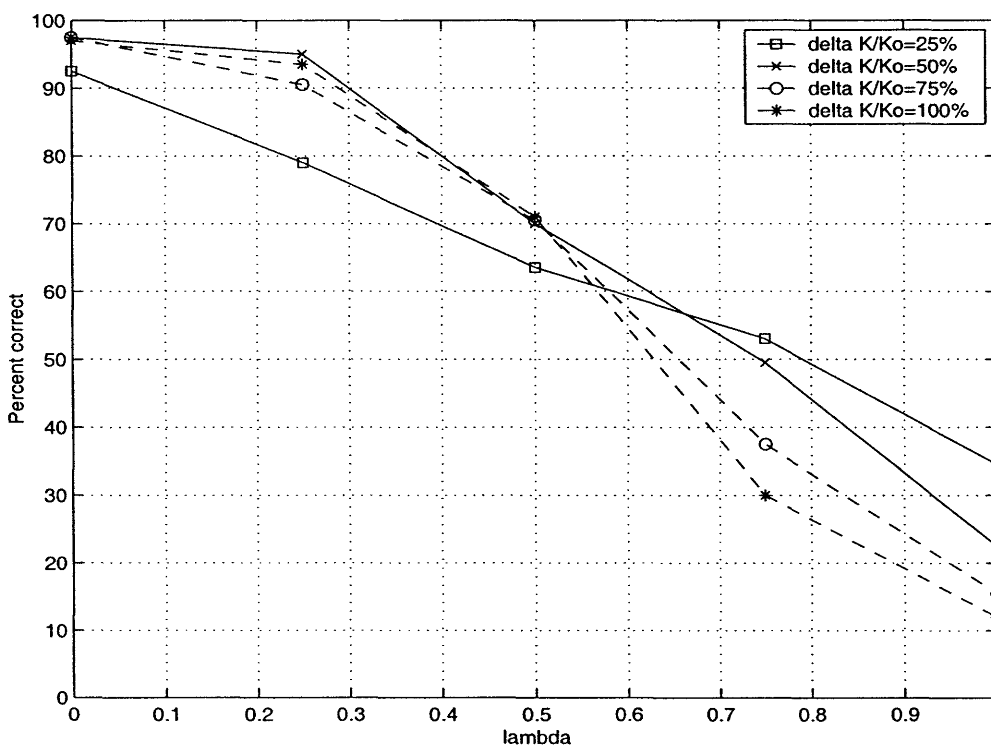


Figure 4.2: Percent correct versus λ in R-F configuration: Front spring standard

Figure 4.3 shows the results for the R-F configuration when the rear spring was the standard. Again, results decreased steadily for all cases of $\Delta K/K_o$ as λ was increased. However, for most values of λ, the percentages of correct responses were consistently lower than for the case in which the front spring was constant. Also, the behavior of the curve for the case in which $\Delta K/K_o$=25% is similar to the corresponding curves in the S-S configuration and in the R-F configuration when the front spring was constant. Between $\lambda=0$ and $\lambda=0.5$ the number of correct responses was significantly lower than the other cases of $\Delta K/K_o$, but it was marginally higher when λ was greater than 0.5.

When $\lambda=0$, subjects responded correctly between 75% to 98% of the cases. For $\lambda=0.25$, correct responses varied from 65.5% to 77.5% across all cases. At $\lambda=0.5$, the percentage of correct responses ranged between 31.5% and 49.5%, while it ranged between 15.5% and 29% when $\lambda=0.75$. Finally, when λ was equal to 1, the subjects responded correctly from 10% to 20% of the cases.

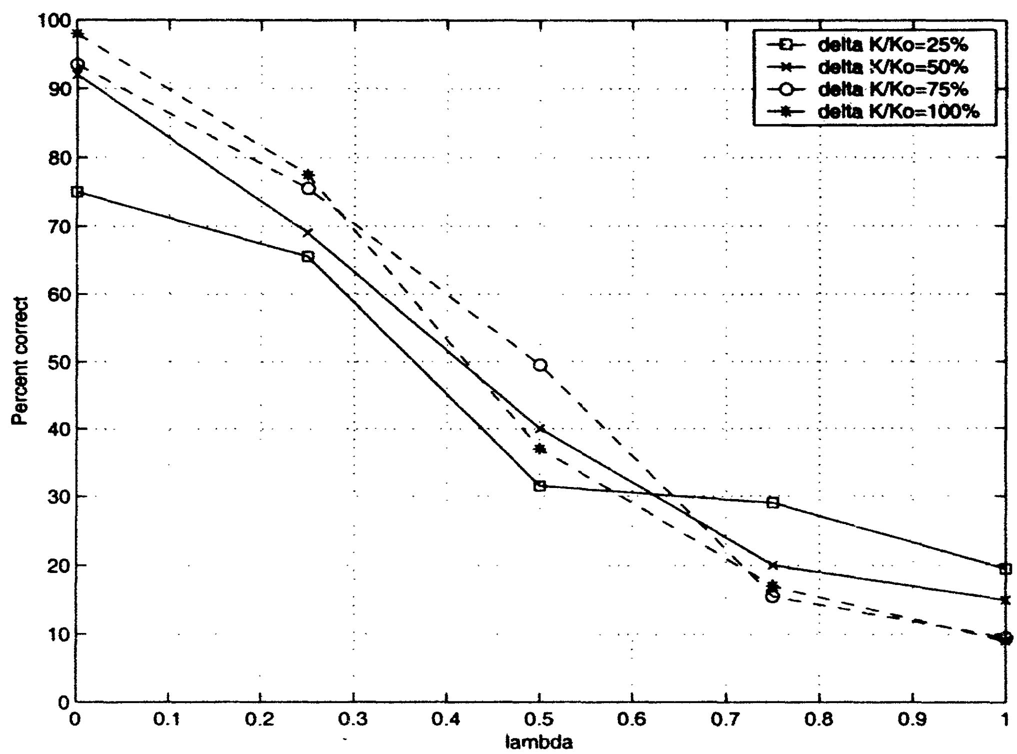


Figure 4.3: Percent correct versus λ in R-F configuration: Rear spring standard

The same results have also been plotted separately for all the different cases of $\Delta K/K_o$. These graphs are shown in figures 4.4 to 4.7. Each graph contains results for the S-S configuration, and both variations of the R-F configuration for their respective value of ΔK. Figure 4.4 shows the results when the difference in stiffness was 25%. A similar graph is shown in Figure 4.5 for the case when the difference in stiffness was 50%. Figures 4.6 and 4.7 show what happened when the stiffness differences were 75% and 100%, respectively. All graphs contain error bars, representing the standard error of the mean.

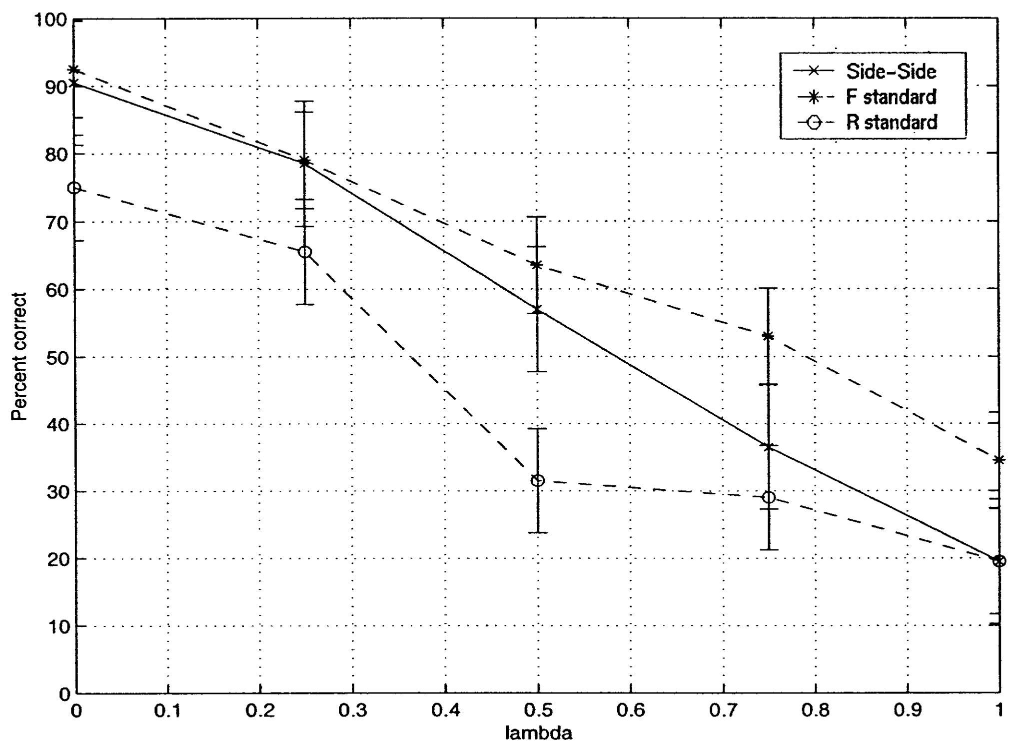


Figure 4.4: Percent correct versus λ when $\Delta K/K_o$=25%

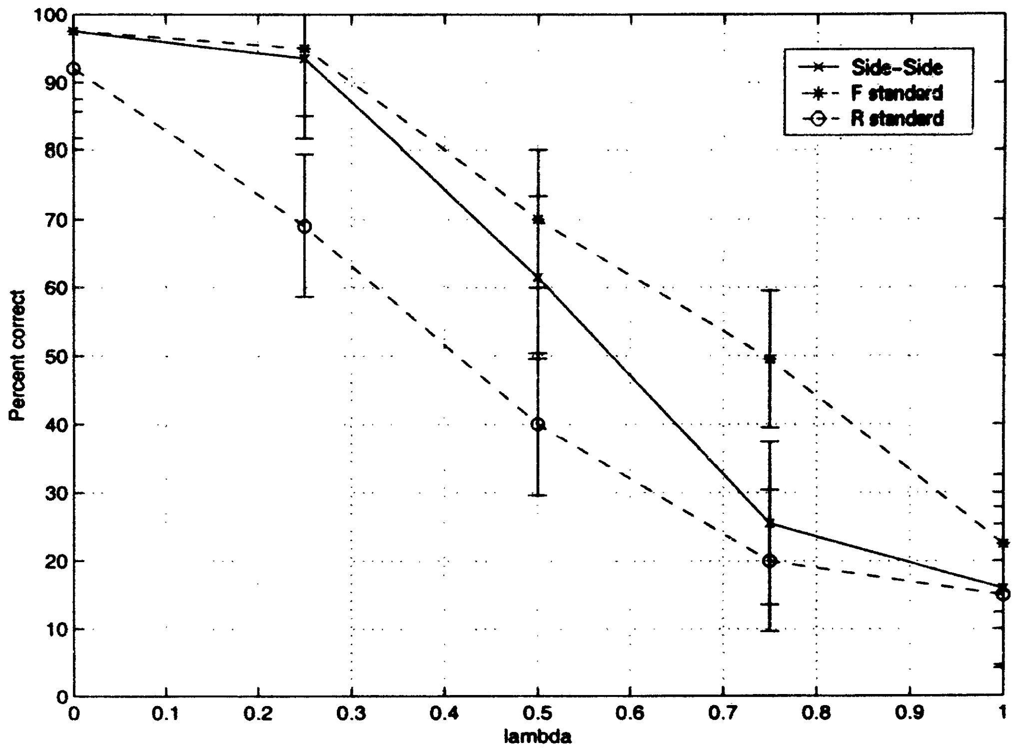


Figure 4.5: Percent correct versus λ when $\Delta K/K_o$=50%

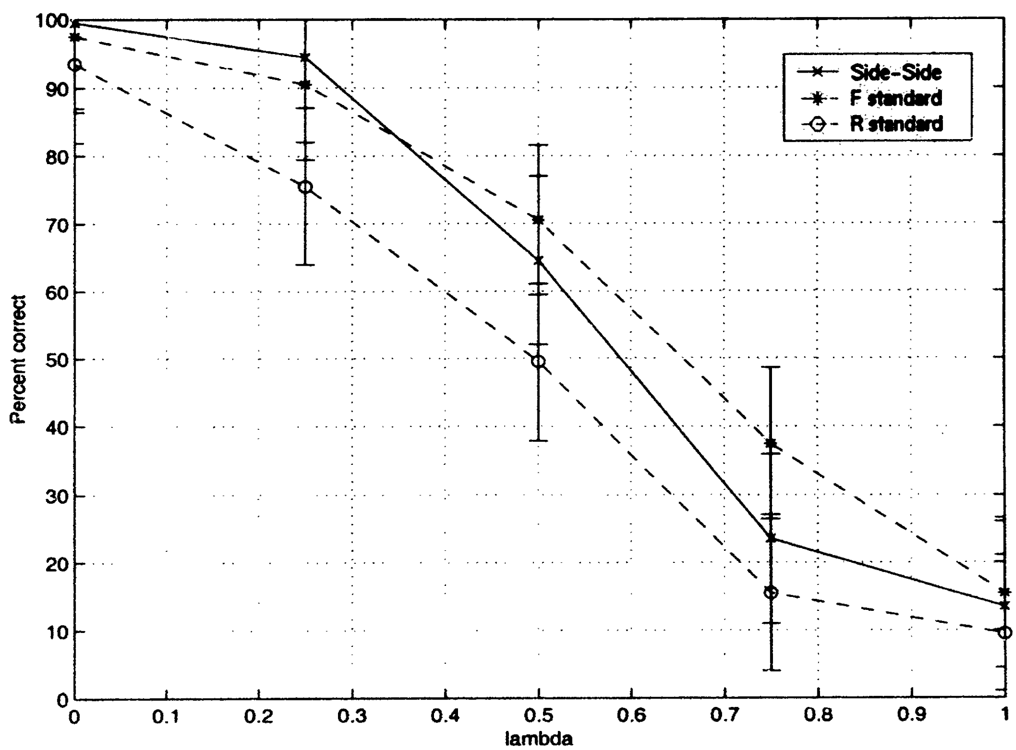


Figure 4.6: Percent correct versus λ when $\Delta K/K_o$=75%

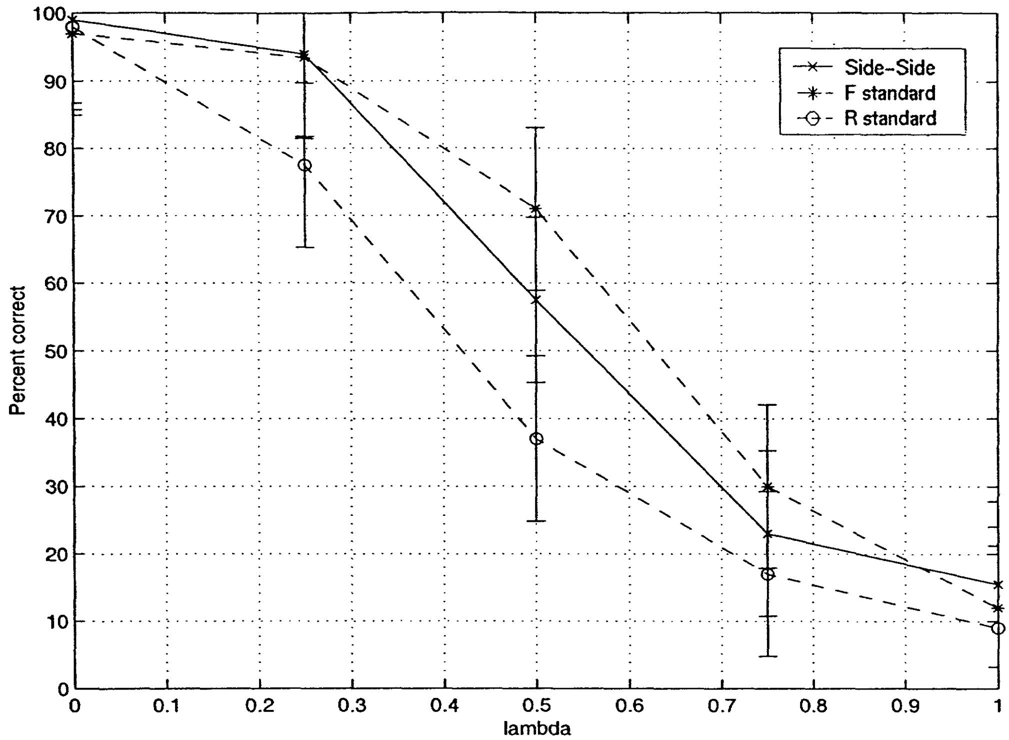


Figure 4.7: Percent correct versus λ when $\Delta K/K_o$=100%

These graphs provide another way of looking at the results, as it can be seen how the number of correct responses varied for the different configurations under the same conditions. For all cases of $\Delta K/K_o$, percentages of correct responses tend to be higher for the R-F configuration when the front spring was standard, followed by the results for the S-S configuration. The results for the R-F configuration when the rear spring was standard tended to be consistently lower than those for the S-S configuration across all values of $\Delta K/K_o$.

Chapter 5

Discussion

Psychophysical experiments were performed for both S-S and R-F configurations with relative stiffness differences between the springs of 25%, 50%, 75%, and 100%. The results of these experiments suggest that the manipulation of visual information can have a strong impact in affecting the perception of mechanical stiffness in virtual environments. The presence of visual dominance over the kinesthetic sense of hand position has become evident, as the percentage of correct responses decreased dramatically as the visual parameter λ increased for all cases of $\Delta K/K_o$ in both configurations. The influence of 3-D perspective can also be described as significant, as for all cases of $\Delta K/K_o$ the number of correct responses when the front spring was the standard in the R-F configuration was higher than the results for the S-S configuration as well as the case when the rear spring was standard in the R-F configuration. Of the three configurations, the R-F configuration with the rear spring as standard consistently provided the lowest number of percent correct responses. Finally, it was also observed that the relationship between the percent of correct responses and λ was not symmetric about the 50% line of correct responses. As $\lambda = 1$ inverted the visual and haptic displacements of the two springs, in general it was expected that for this value of λ the percent of correct responses would tend to go to zero. However, on average this value approximated 10% and not zero, while for $\lambda = 0$ the number of correct responses was close to 100% as expected.

The results obtained for both S-S and R-F configurations are very consistent, taking into account that ten subjects participated in the study. In order to explain the results, a concept we define as "apparent stiffness" must be introduced. The apparent stiffness of a spring is defined as the actual applied force divided by the visual displacement of the spring:

$$K_{apparent_{ref}} = \frac{F}{x_{v,\,ref}} \tag{5.1}$$

$$K_{apparent_{comp}} = \frac{F}{x_{v,\,comp}} \tag{5.2}$$

By substituting equation 3.2 into equation 5.1, and equation 3.4 into equation 5.2, the following expressions are obtained for the apparent stiffness:

$$K_{apparent_{ref}} = (1-\lambda)K_o + \lambda(K_o + \Delta K) \tag{5.3}$$

$$K_{apparent_{comp}} = (1-\lambda)(K_o + \Delta K) + \lambda K_o \tag{5.4}$$

where $K_{apparent_{ref}}$ is the apparent stiffness for the standard spring (the one with the true reference stiffness K_o) and $K_{apparent_{comp}}$ is the apparent stiffness of the comparison spring.

Using these equations, the apparent stiffness was calculated for both springs in the S-S configuration in order to obtain a possible explanation of what the subjects were perceiving under the influence of manipulated visual information. In the R-F configuration the apparent stiffness was also calculated for both springs. However, equations 5.3 and 5.4 needed to be modified to account for the effect of 3-D perspective. A proportionality constant of 1.33 was measured between the deformation of the front spring against that of the rear one. As the rear spring seems visually smaller and thus seems to deform less than what it actually should, the proportionality constant needed to be incorporated into equations 5.3 and 5.4. Thus, apparent stiffness for both springs in both cases of the R-F configuration, where the front and rear springs are standard, is given by the following set of equations:

$$K_{apparent_{F,\,ref}} = (1-\lambda)K_o + \lambda(K_o + \Delta K) \tag{5.5}$$

$$K_{apparent_{F,\,comp}} = 1.33(1-\lambda)(K_o + \Delta K) + 1.33\lambda K_o \tag{5.6}$$

$$K_{apparent_{R,ref}} = 1.33(1-\lambda)K_o + 1.33\lambda(K_o + \Delta K) \quad (5.7)$$

$$K_{apparent_{R,comp}} = (1-\lambda)(K_o + \Delta K) + \lambda K_o \quad (5.8)$$

where $K_{apparent_{F,ref}}$ and $K_{apparent_{F,comp}}$ are the apparent stiffness for the standard and comparison springs when the front spring is the standard, and $K_{apparent_{R,ref}}$ and $K_{apparent_{R,comp}}$ the respective apparent stiffness for the two springs when the rear spring is the standard. Notice that equation 5.5 for $K_{apparent_{F,ref}}$ and equation 5.8 for $K_{apparent_{R,comp}}$ remained the same as equations 5.3 and 5.4, respectively.

The experiment with the S-S configuration only measured the effects that manipulated visual information might have on the perception of stiffness, and did not include the effects of 3-D perspective. In this case, if subjects had discriminated solely on the basis of haptic force and displacement of their hand, and ignored all the provided visual information, then the subjects should have obtained nearly 100% correct, regardless of the value of λ.

However, the results for both S-S and R-F show that the subjects discriminated between the stiffness of the two springs by taking into account both the visual and haptic information provided. In order to predict what the behavior would be if the subjects had responded based purely on the apparent stiffness, apparent stiffness values were calculated for both springs for all cases of $\Delta K/K_o$. The difference between the apparent stiffness of reference and companion springs was plotted as a function of λ in order to find a relationship to the results presented in the graphs shown in figures 4.4 to 4.7. Similarly, the percent of change in apparent stiffness difference (difference in apparent stiffness divided by the apparent stiffness of the reference spring) was plotted against λ to look for similar trends. All the values for apparent stiffness, apparent stiffness difference, and percent of change in apparent stiffness difference have been tabulated in Appendix B. Values for apparent stiffness difference, and percent of change in apparent stiffness difference were plotted systematically across all values of $\Delta K/K_o$. For each case of $\Delta K/K_o$, the obtained values for the S-S configuration, R-F configuration with the front spring standard, and R-F configuration with the rear spring standard were plotted in the same graph in order to observe the differences between them.

Figure 5.1 shows the experimental results for the case when $\Delta K/K_o$=25%. This graph was already presented in figure 4.4, but is shown again so the similarities in the trends with the graphs of apparent stiffness difference and percent of change in apparent stiffness difference versus λ can be observed. Figure 5.2 shows the difference between the apparent stiffness of the two springs as a function of λ for the particular case when $\Delta K/K_o$=25%. The graph on figure 5.3 shows the percent of change in apparent stiffness difference as a function of λ.

Figures 5.4 to 5.6 show the same set of graphs for the case when $\Delta K/K_o$=50%, while figures 5.7 to 5.9 show the same results when $\Delta K/K_o$=75% and figures 5.10 to 5.12 show the same when $\Delta K/K_o$=100%.

It was found that for the four values of $\Delta K/K_o$, there is indeed a strong relationship between the experimental results and the predicted behavior in terms of apparent stiffness. By observing the graphs showing the apparent stiffness difference versus λ, it can be seen that the percentage of correct responses was expected to decrease monotonically as the visual scaling parameter λ was increased. The effects of 3-D perspective are also captured in these graphs, as the percent of correct responses was expected to be systematically higher across values of λ for the R-F configuration when the front spring was constant. Also, the number of correct responses in the S-S configuration was expected to be higher than in the R-F case when the rear spring was constant. This behavior can be observed in the graphs showing the experimental results for all values of $\Delta K/K_o$. However, the expected behavior in terms of apparent stiffness differences is linear and skew-symmetric about the horizontal axis. The experimental results do not show this complete linearity or symmetry about the 50% correct horizontal axis. For all cases of $\Delta K/K_o$, the experimental curves seem to be flatter between $\lambda=0$ and $\lambda=0.25$ and approach 100% correct responses. This can be justified, as 100% is the maximum percentage of correct responses that is possible. On the other hand, as λ approaches 1, the results seem to approach 10% of correct responses instead of the expected 0%. This behavior is very consistent for both configurations. However, the graphs of percent of change in apparent stiffness difference as a function of λ do capture this lack of symmetry about the horizontal axis (Figures 5.3, 5.6, 5.9, and 5.12). This suggests that the presence of lack of symmetry in the curves for experimental results is not a coincidence, but needs to be studied further.

Another observation made from the graphs is that as $\Delta K/K_o$ increases, the slope of the apparent stiffness difference curves increases. In the experimental results, it can be observed how for the $\Delta K/K_o$=25% the behavior seemed to be flatter. This observation is best appreciated in figures 4.1 to 4.3. This behavior suggests how the impact of manipulated visual information is stronger for larger values of ΔK, and is another area that needs further study.

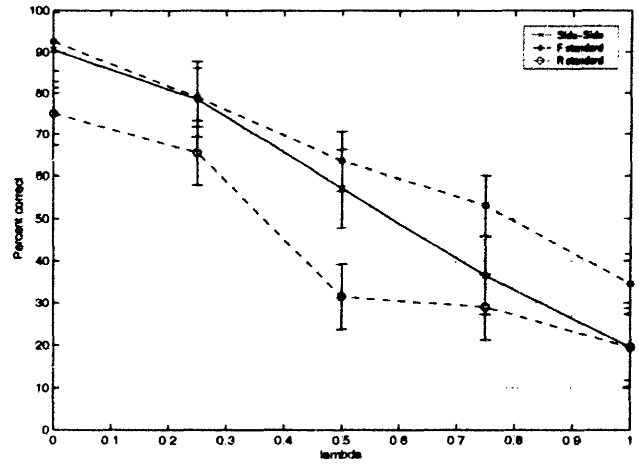


Figure 5.1: Percent correct versus λ when $\Delta K/K_o$=25%

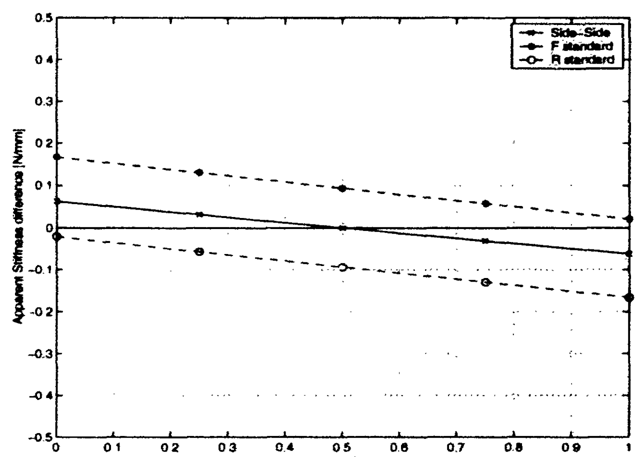


Figure 5.2: Apparent stiffness difference versus λ when $\Delta K/K_o$=25%

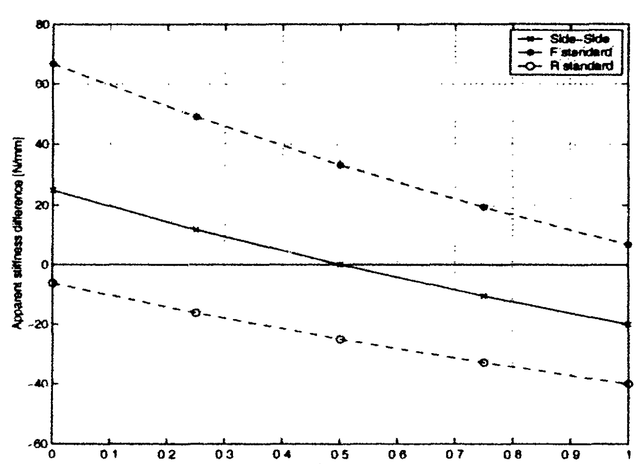


Figure 5.3: Percent of change in apparent stiffness difference when $\Delta K/K_o$=25%

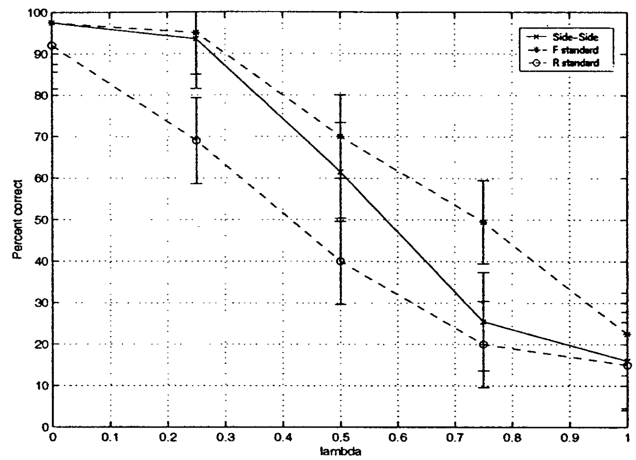


Figure 5.4: Percent correct versus λ when $\Delta K/K_o$=50%

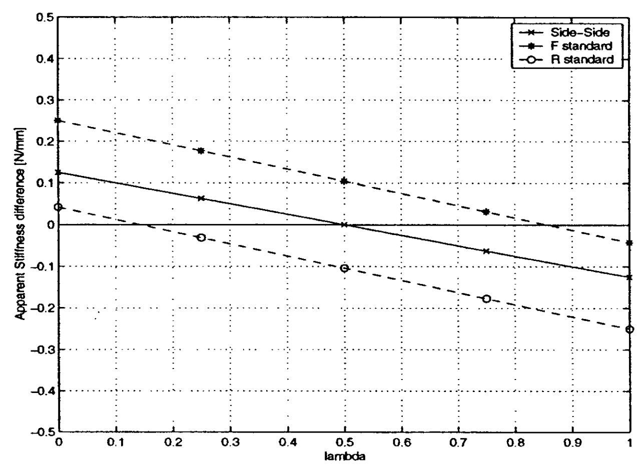


Figure 5.5: Apparent stiffness difference versus λ when $\Delta K/K_o$=50%

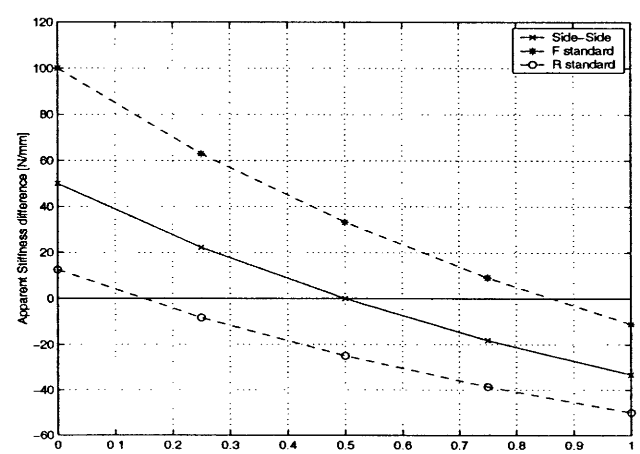


Figure 5.6: Percent of change in apparent stiffness difference when $\Delta K/K_o$=50%

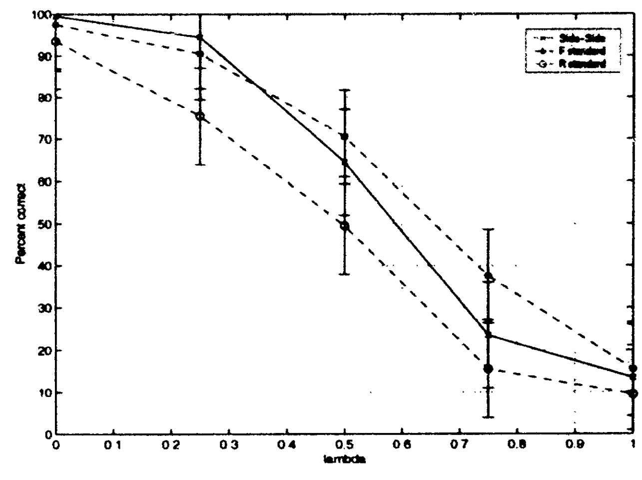


Figure 5.7: Percent correct versus λ when $\Delta K/K_o$=75%

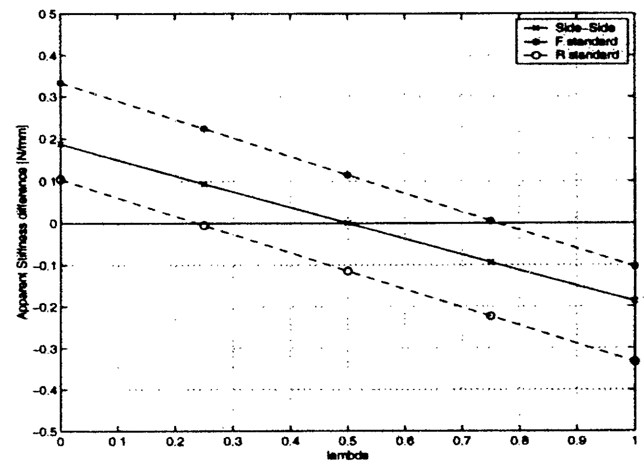


Figure 5.8: Apparent stiffness difference versus λ when $\Delta K/K_o$=75%

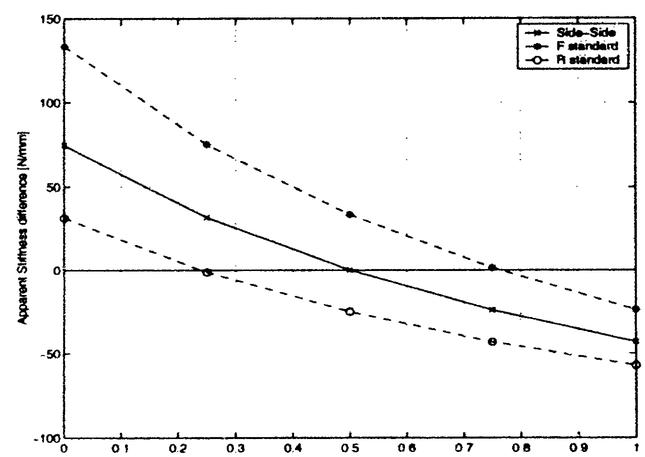


Figure 5.9: Percent of change in apparent stiffness difference when $\Delta K/K_o$=75%

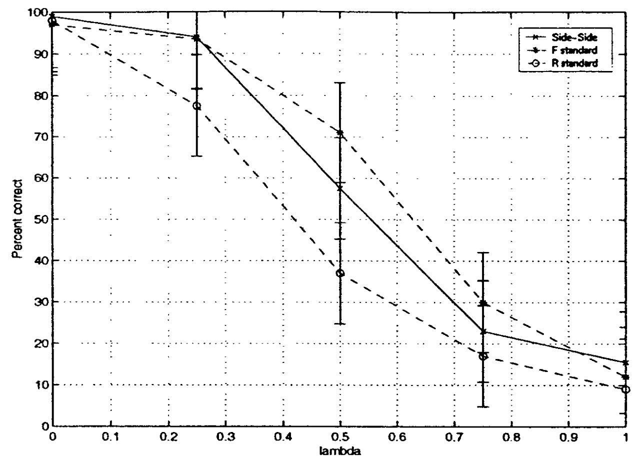


Figure 5.10: Percent correct versus λ when $\Delta K/K_o$=100%

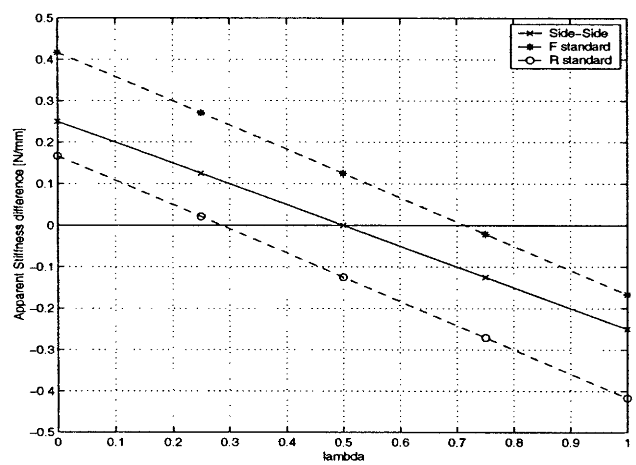


Figure 5.11: Apparent stiffness difference versus λ when $\Delta K/K_o$=100%

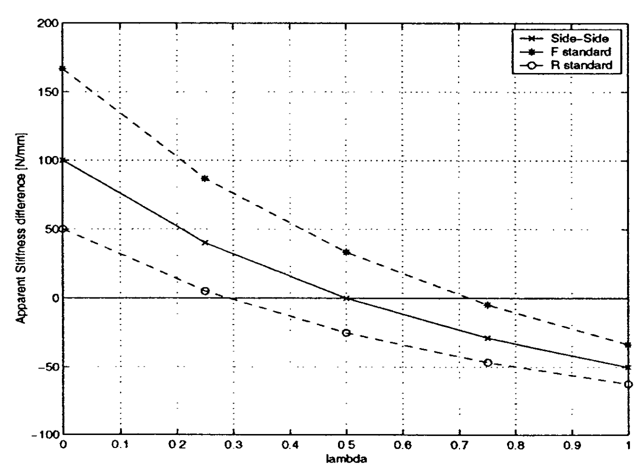


Figure 5.12: Percent of change in apparent stiffness difference when $\Delta K/K_o$=100%

Chapter 6

Conclusions

Psychophysical experiments have been conducted on ten different subjects using the Phantom haptic interface in order to determine if manipulated visual information and 3-D perspective can influence the haptic perception of stiffness in virtual environments.

It was found that visual information has a clear dominance over the kinesthetic hand position information in the discrimination of stiffness in this type of virtual environmments. This dominance became more evident as the visual scaling parameter was increased. In addition, 3-D visual perspective adds an important contribution to these illusions, making objects that are located farther away be perceived to be stiffer by the subjects. This can be concluded since for all cases of $\Delta K/K_o$ the number of correct responses when the front spring was the standard in the R-F configuration was higher than the S-S configuration and the case when the rear spring was standard in the R-F configuration. Of the three configurations, the R-F configuration with the rear spring as standard consistently provided the lowest number of correct responses, suggesting that the 3-D perspective makes objects that are farther away to be perceived as stiffer.

Finally, it was also observed that the relationship between the percent of correct responses and λ was not symmetric about the 50% line of correct responses. This lack of symmetry was also observed in the graphs showing the percentage of apparent stiffness difference versus λ, and is an area in which further research needs to be done.

Further studies can also be designed in order to find up to what limits can manipulated visual information be incorporated in visual environments in order to enhance the haptic experience. It is believed that the visual scaling factor will reach a value where the illusions will break down when the visual display may seem unrelated to the haptic experience.

In general, results from this study suggest that a proper combination of manipulated visual information and illusions created by 3-D perspective can strongly influence the haptic perception of stiffness in virtual environments. Thus, by providing a proper relationship between haptic and visual information, and the addition of 3-D perspective, the haptic experience in mutimodal virtual environments can be improved, overcoming the limitations of current haptic interfaces.

Appendix A

Data Tables with Subject Responses

A.1 Subject Responses in Side - Side Configuration

	$\lambda=0$	$\lambda=0.25$	$\lambda=0.5$	$\lambda=0.75$	$\lambda=1.0$
Subject 1	90%	70%	45%	40%	15%
Subject 2	95%	75%	60%	15%	50%
Subject 3	95%	80%	70%	45%	5%
Subject 4	75%	80%	75%	35%	20%
Subject 5	90%	75%	40%	25%	15%
Subject 6	90%	80%	75%	65%	15%
Subject 7	80%	80%	45%	35%	20%
Subject 8	95%	90%	65%	60%	40%
Subject 9	100%	75%	45%	45%	5%
Subject 10	95%	80%	50%	0%	10%
Average	90.5%	78.5%	57%	36.5%	19.5%

Table A.1: Percent correct versus λ when $\Delta K/K_o$=25% (S-S)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	95%	90%	45%	30%	0%
Subject 2	100%	100%	80%	30%	25%
Subject 3	100%	95%	60%	20%	5%
Subject 4	95%	95%	70%	50%	45%
Subject 5	100%	95%	55%	25%	0%
Subject 6	95%	85%	100%	25%	20%
Subject 7	95%	90%	55%	20%	0%
Subject 8	95%	95%	70%	50%	55%
Subject 9	100%	100%	50%	5%	5%
Subject 10	100%	90%	30%	0%	5%
Average	97.5%	93.5%	61.5%	25.5%	16%

Table A.2: Percent correct versus λ when $\Delta K/K_o$=50% (S-S)

	$\lambda=0$	$\lambda=0.25$	$\lambda=0.5$	$\lambda=0.75$	$\lambda=1.0$
Subject 1	100%	95%	65%	5%	0%
Subject 2	100%	100%	80%	30%	25%
Subject 3	100%	100%	75%	30%	5%
Subject 4	100%	85%	80%	75%	45%
Subject 5	100%	90%	55%	5%	10%
Subject 6	100%	95%	75%	25%	0%
Subject 7	100%	90%	55%	25%	0%
Subject 8	95%	95%	70%	40%	50%
Subject 9	100%	95%	50%	0%	0%
Subject 10	100%	100%	40%	0%	0%
Average	99.5%	94.5%	64.5%	23.5%	13.5%

Table A.3: Percent correct versus λ when $\Delta K/K_o$=75% (S-S)

	$\lambda=0$	$\lambda=0.25$	$\lambda=0.5$	$\lambda=0.75$	$\lambda=1.0$
Subject 1	100%	100%	60%	15%	0%
Subject 2	100%	95%	70%	35%	10%
Subject 3	95%	75%	60%	15%	50%
Subject 4	95%	95%	80%	70%	20%
Subject 5	100%	95%	40%	5%	0%
Subject 6	100%	95%	60%	15%	10%
Subject 7	100%	95%	50%	15%	5%
Subject 8	100%	95%	65%	45%	50%
Subject 9	100%	100%	60%	15%	10%
Subject 10	100%	95%	25%	0%	0%
Average	99%	94%	57.5%	23%	15.5%

Table A.4: Percent correct versus λ when $\Delta K/K_o$=100% (S-S)

A.2 Subject Responses in R - F Configuration: Front Spring Standard

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	95%	95%	65%	45%	35%
Subject 2	90%	75%	60%	70%	70%
Subject 3	90%	90%	85%	80%	55%
Subject 4	90%	70%	65%	70%	25%
Subject 5	100%	70%	65%	60%	30%
Subject 6	100%	80%	65%	60%	35%
Subject 7	95%	80%	75%	45%	45%
Subject 8	80%	65%	70%	55%	30%
Subject 9	85%	75%	40%	20%	5%
Subject 10	100%	90%	45%	25%	15%
Average	92.5%	79%	63.5%	53%	34.5%

Table A.5: Percent correct versus λ when $\Delta K/K_o$=25% (F Standard)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	100%	90%	80%	45%	15%
Subject 2	100%	95%	90%	60%	20%
Subject 3	95%	100%	75%	90%	35%
Subject 4	100%	95%	75%	65%	5%
Subject 5	100%	100%	75%	65%	70%
Subject 6	100%	100%	75%	45%	20%
Subject 7	100%	100%	50%	45%	5%
Subject 8	85%	95%	85%	65%	50%
Subject 9	100%	95%	40%	10%	0%
Subject 10	100%	95%	40%	10%	0%
Average	97.5%	95%	70%	49.5%	22.5%

Table A.6: Percent correct versus λ when $\Delta K/K_o$=50% (F Standard)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	100%	95%	65%	20%	10%
Subject 2	100%	100%	85%	60%	20%
Subject 3	95%	100%	95%	75%	40%
Subject 4	100%	90%	75%	65%	30%
Subject 5	100%	90%	80%	25%	5%
Subject 6	95%	100%	70%	45%	15%
Subject 7	100%	95%	60%	20%	0%
Subject 8	90%	60%	45%	45%	35%
Subject 9	100%	95%	60%	10%	0%
Subject 10	95%	80%	70%	10%	0%
Average	97.5%	90.5%	70.5%	37.5%	15.5%

Table A.7: Percent correct versus λ when $\Delta K/K_o$=75% (F Standard)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	100%	100%	65%	20%	5%
Subject 2	100%	90%	85%	50%	25%
Subject 3	95%	100%	85%	55%	0%
Subject 4	95%	80%	95%	50%	45%
Subject 5	100%	100%	75%	20%	10%
Subject 6	90%	100%	80%	25%	5%
Subject 7	95%	95%	70%	20%	0%
Subject 8	95	85%	80%	45%	25%
Subject 9	100%	95%	30%	5%	0%
Subject 10	100%	90%	45%	10%	5%
Average	97%	93.5%	71%	30%	12%

Table A.8: Percent correct versus λ when $\Delta K/K_o$=100% (F Standard)

A.3 Subject Responses in R - F Configuration: Rear Spring Standard

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	70%	65%	15%	20%	10%
Subject 2	90%	95%	50%	35%	50%
Subject 3	75%	50%	30%	30%	10%
Subject 4	45%	55%	35%	45%	40%
Subject 5	55%	45%	25%	10%	15%
Subject 6	75%	65%	30%	40%	25%
Subject 7	95%	50%	45%	20%	5%
Subject 8	60%	55%	25%	45%	35%
Subject 9	90%	90%	30%	30%	5%
Subject 10	95%	85%	30%	15%	0%
Average	75%	65.5%	31.5%	29%	19.5%

Table A.9: Percent correct versus λ when $\Delta K/K_o$=25% (R Standard)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	90%	45%	35%	30%	15%
Subject 2	95%	85%	60%	25%	50%
Subject 3	85%	70%	35%	10%	0%
Subject 4	80%	50%	60%	25%	30%
Subject 5	85%	75%	5%	15%	5%
Subject 6	95%	70%	40%	20%	10%
Subject 7	100%	65%	35%	10%	5%
Subject 8	90%	60%	45%	45%	35%
Subject 9	100%	80%	55%	10%	0%
Subject 10	100%	90%	30%	10%	0%
Average	92%	69%	40%	20%	15%

Table A.10: Percent correct versus λ when $\Delta K/K_o$=50% (R Standard)

	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1.0
Subject 1	85%	80%	20%	5%	10%
Subject 2	95%	90%	70%	45%	25%
Subject 3	90%	40%	25%	10%	0%
Subject 4	90%	80%	65%	15%	20%
Subject 5	100%	55%	20%	5%	0%
Subject 6	100%	70%	50%	5%	0%
Subject 7	100%	90%	60%	15%	5%
Subject 8	80%	80%	60%	40%	30%
Subject 9	100%	90%	55%	5%	5%
Subject 10	95%	80%	70%	10%	0%
Average	93.5%	75.5%	49.5%	15.5%	9.5%

Table A.11: Percent correct versus λ when $\Delta K/K_o$=75% (R Standard)

	$\lambda=0$	$\lambda=0.25$	$\lambda=0.5$	$\lambda=0.75$	$\lambda=1.0$
Subject 1	100%	65%	20%	15%	0%
Subject 2	95%	85%	60%	35%	30%
Subject 3	95%	85%	25%	5%	0%
Subject 4	95%	65%	60%	30%	20%
Subject 5	100%	60%	10%	5%	5%
Subject 6	100%	75%	25%	10%	0%
Subject 7	100%	90%	50%	10%	5%
Subject 8	95%	70%	60%	45%	20%
Subject 9	100%	85%	30%	15%	5%
Subject 10	100%	95%	30%	0%	5%
Average	98%	77.5%	37%	17%	9%

Table A.12: Percent correct versus λ when $\Delta K/K_o$=100% (R Standard)

Appendix B

Data Tables with Apparent Stiffness of Springs

B.1 Apparent Stiffness in S-S Configuration

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.3125	0.06	25%
0.25	0.265625	0.296875	0.03	12%
0.5	0.28125	0.28125	0	0%
0.75	0.296875	0.265625	-0.03	-11%
1.0	0.3125	0.25	-0.06	-20%

Table B.1: Apparent stiffness of springs when $\Delta K/K_o$=25% in S-S

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.375	0.13	50%
0.25	0.28125	0.34375	0.06	22%
0.5	0.3125	0.3125	0	0%
0.75	0.34375	0.28125	-0.06	-18%
1.0	0.375	0.25	-0.13	-33%

Table B.2: Apparent stiffness of springs when $\Delta K/K_o$=50% in S-S

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.4375	0.19	75%
0.25	0.296875	0.390625	0.09	32%
0.5	0.34375	0.34375	0	0%
0.75	0.390625	0.296875	-0.09	-24%
1.0	0.4375	0.25	-0.19	-43%

Table B.3: Apparent stiffness of springs when $\Delta K/K_o$=75% in S-S

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.5	0.25	100%
0.25	0.3125	0.4375	0.13	40%
0.5	0.375	0.375	0	0%
0.75	0.4375	0.3125	-0.13	-29%
1.0	0.5	0.25	-0.25	-50%

Table B.4: Apparent stiffness of springs when $\Delta K/K_o$=100% in S-S

B.2 Apparent Stiffness in R-F Configuration (F Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.416667	0.17	67%
0.25	0.265625	0.395833	0.13	49%
0.5	0.28125	0.375	0.09	33%
0.75	0.296875	0.3541667	0.06	19%
1.0	0.3125	0.3333	0.02	7%

Table B.5: Apparent stiffness of springs when $\Delta K/K_o$=25% in R-F (F Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.5		100%
0.25	0.28125	0.458333		63%
0.5	0.3125	0.416667		33%
0.75	0.34375	0.375		9%
1.0	0.375	0.3333		-11%

Table B.6: Apparent stiffness of springs when $\Delta K/K_o$=50% in R-F (F Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.58333	0.33	133%
0.25	0.296875	0.520833	0.22	75%
0.5	0.34375	0.458333	0.11	33%
0.75	0.390625	0.395833	0.01	1%
1.0	0.4375	0.3333	-0.10	-24%

Table B.7: Apparent stiffness of springs when $\Delta K/K_o$=75% in R-F (F Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.25	0.6667	0.42	167%
0.25	0.3125	0.58333	0.27	87%
0.5	0.375	0.5	0.125	33%
0.75	0.4375	0.41667	-0.02	-5%
1.0	0.5	0.333	-0.17	-33%

Table B.8: Apparent stiffness of springs when $\Delta K/K_o$=100% in R-F (F Standard)

B.3 Apparent Stiffness in R-F Configuration (R Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.3333	0.3125	-0.02	-6%
0.25	0.3541667	0.296875	-0.06	-16%
0.5	0.375	0.28125	-0.09	-25%
0.75	0.395833	0.265625	-0.13	-33%
1.0	0.41667	0.25	-0.17	-40%

Table B.9: Apparent stiffness of springs when $\Delta K/K_o$=25% in R-F (R Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.3333	0.375	0.04	13%
0.25	0.375	0.34375	-0.03	-8%
0.5	0.41667	0.3125	-0.10	-25%
0.75	0.45833	0.28125	-0.18	-39%
1.0	0.5	0.25	-0.25	-50%

Table B.10: Apparent stiffness of springs when $\Delta K/K_o$=50% in R-F (R Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.3333	0.4375	0.10	31%
0.25	0.395933	0.390625	-0.01	-1%
0.5	0.45833	0.34375	-0.11	-25%
0.75	0.520833	0.296875	-0.22	-43%
1.0	0.58333	0.25	-0.33	-57%

Table B.11: Apparent stiffness of springs when $\Delta K/K_o$=75% in R-F (R Standard)

λ	Apparent Stiffness (Standard) [N/mm]	Apparent Stiffness (Comparison) [N/mm]	Apparent Stiffness Difference [N/mm]	Percent of Difference
0	0.3333	0.5	0.17	50%
0.25	0.41667	0.4375	0.02	5%
0.5	0.5	0.375	-0.13	-25%
0.75	0.58333	0.3125	-0.27	-46%
1.0	0.66667	0.25	-0.42	-63%

Table B.12: Apparent stiffness of springs when $\Delta K/K_o$=100% in R-F (R Standard)

References

[1] Burdea, G.C. 1996, *Force and Touch Feedback for Virtual Reality*, John Wiley and Sons, Inc.

[2] Durfee, W. K., Hendrix, C.M., Cheng, P., Varughese, G., 1997, "Influence of Haptic and Visual Displays on the Estimation of Virtual Environment Stiffness", *ASME Dynamic Systems and Control Division*, DSC-Vol. 61, pp. 139-144, Fairfield, New Jersey.

[3] Gliner, C. R., Pick, A.D., Pick, H. L., Hales, J.J., September 1969, "A Developmental Investigation of Visual and Haptic Preferences for Shape and Texture", *Monographs of the Society for Research in Child Development*, Serial No. 130, Vol. 34, No. 6.

[4] Heller, M. A., 1982, "Visual and Tactual Texture Perception: Intersensory Cooperation", *Perception & Psychophysics*, Vol. 31, pp. 339-344.

[5] Heller, M. A., Schiff, W., 1991, *The Psychology of Touch*, Lawrence Erlbaum Associates, Inc.

[6] Kinney, J. A., Luria, S. M., 1970, "Conflicting Visual and Tactual-Kinesthetic Stimulation", *Perception & Psychophysics*, Vol. 8, pp. 189-192.

[7] Lederman, S. J,. Taylor, M. M., 1969, "Perception of Interpolated Position and Orientation by Vision and Active Touch", *Perception & Psychophysics*, Vol. 6, pp. 153-159.

[8] McDonald, L., Vince, J., 1994, *Interacting with Virtual Environments*, John Wiley and Sons, Inc.

[9] Rosenberg, L. B., Adelstein, B. D., 1993, "Perceptual Decomposition of Virtual Haptic Surfaces", *Proceedings of IEEE Symposium on Research Frontiers in Virtual Reality*, pp. 46-53, San Jose, California.

[10] Salisbury, J. K., Srinivasan, M. A., Sept. - Oct. 1997, "PHANToM Based Haptic Interaction with Virtual Objects", *IEEE Computer Graphics and Applications.*

[11] Srinivasan, M. A., 1995, "Haptic Interfaces", in Durlach, N. I., and Mavor, A. S. (Eds.), *Report of the Committee on Virtual Reality Research and Development*, Chap. 4, National Research Council, National Academy Press, Washington, D.C.

[12] Srinivasan, M. A., Basdogan, Cagatay, 1997, "Haptics in Virtual Environments: Taxonomy, Research Status, and Challenges", *Computers and Graphics*, Vol. 21, No. 4.

[13] Srinivasan, M. A., Beauregard, G.L., Brock, D.L., 1996, "The Impact of Visual Information on Haptic Perception of Stiffness in Virtual Environments", *Proceedings of ASME Dynamic Systems and Control Division*, DSC-Vol. 58, pp. 555-559, Atlanta, Georgia.

[14] Stuart, R., 1996, *The Design of Virtual Environments*, McGraw-Hill.

[15] Tan, H. Z., 1997, "The Identification of Sphere Size Using the PHANToM: Towards a Set of Building Blocks for Rendering Haptic Environment", *Proceedings of ASME Winter Annual Meeting.*

[16] Warren, D.H., and Welch, R.B., 1986, "Intersensory Interactions", *Handbook of Perception and Human Performance, Vol. I, Sensory Processes and Perception, Chapter 25*, Wiley and Sons.

[17] Weisenberger, J. M., Krier, M. J., 1997, "Haptic Perception of Simulated Surface Textures via Vibratory and Force Feedback Displays", *ASME Dynamic Systems and Control Division*, DSC-Vol. 61, pp. 55-60, Fairfield, New Jersey.

[18] Wu, W. C., Srinivasan, M.A., and Basdogan, C., 1999, "Visual, Haptic, and Bimodal Perception of Size and Stiffness in Virtual Environments", *ASME Dynamic Systems and Control Division*, DSC-Vol. 67.